# Bhagalpur College of Engineering, Bhagalpur

Model Question Paper

Branch – Civil Engineering

## B.Tech 1st Semester Exam, 2023
## (New course)

MATHEMATICS – I

## (Calculus and Linear Algebra)

Time : 3 hours | Full Marks : 70

Instruction:
i) The marks are indicated in the right-hand margin.
ii) There are **NINE** question in this paper.
iii) Attempt **FIVE** question in all.
iv) Question No. **1** is Compulsory.

**1.** Choose the correct answer (any seven) :

2×7=14

(a) If Y = $\int_0^{\pi} \log \sin x \, dx$, then the value of Y is

(i) $-\pi \log 2$ (ii) $\pi \log 2$

(iii) $\log 2$ (iv) $-\log 2$

**(Turn over)**

(b) The value of $\Gamma(\frac{1}{2})$ is

(i) $\pi$ (ii) $\sqrt{\pi}$

(iii) $2\sqrt{\pi}$ (iv) None

(c) The area of a loop of the curve r = asin3Θ, is

(i) $\frac{\pi a^2}{12}$ (ii) $\frac{\pi}{12}$

(iii) $\frac{a^2}{12}$ (iv) None

(d) The value of $\lim_{x \to 0} \mathrm{Sinx} \log x$ is

(i) 1 (ii) 0

(iii) 2 (iv) ∞

(e) The series

$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} \dots \dots \dots \dots . \infty$ is convergent for

(i) $p \geq 1$ (ii) $p < 1$

(iii) $p > 1$ (iv) $p = 1$

**(Turn over)**

(f) If $\sum u_n = \sum \frac{1}{n^n}$ , $\sum v_n = \sum \frac{1}{(\log n)^n}$ , then

(i) $\sum u_n$ convergent but $\sum v_n$ divergent.
(ii) $\sum u_n$ divergent but $\sum v_n$ convergent.
(iii) $\sum u_n$ and $\sum v_n$ both convergent.
(iv) $\sum u_n$ and $\sum v_n$ both divergent.

(g) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ is

(i) 0 (ii) 1

(iii) -1 (iv)Does not exist

(h) Gradient of the function
$Q = \log(x^2 + y^2 + z^2)$ is

(i) $\frac{2(x\hat{i}+y\hat{j}+z\acute{k})}{x^2+y^2+z^2}$ (ii) $\frac{(x\hat{i}+y\hat{j}+z\acute{k})}{x^2+y^2+z^2}$

(iii) $\frac{(x\hat{i}+y\hat{j}+z\acute{k})}{x+y+z}$ (iv) $\frac{2(x\hat{i}+y\hat{j}+z\acute{k})}{x+y+z}$

(i) Conjugate of a matrix

$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix} \text{ is}$$

(i) $\bar{A} = \begin{bmatrix} -1-i & -2+3i & -4 \\ -7-2i & i & -3+2i \end{bmatrix}$

**(Turn over)**

(ii) $\bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$

(iii) $\bar{A} = \begin{bmatrix} 1-i & 2-3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$

(iv) $\bar{A} = \begin{bmatrix} 1 & 2 & 4 \\ 7 & 0 & 3 \end{bmatrix}$

(j) Rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix} \text{ is}$$

(i) 2      (ii) 1

(iii) 3      (iv) 0

**2.** (a) Evaluate $\int_0^\infty e^{-ax} x^{m-1} \text{Sinbx}\, dx$ in terms of Gamma function.

(b) Show that the area between the parabolas $y^2$=4ax and $x^2$=4ay is $\frac{16}{3}a^2$.

7+7

**3.** (a) Prove that the equation $2x^3$-3x$^2$-x+1 = 0 has at least one root between 1 and 2.

(b) Evaluate $\lim_{x \to 0} \frac{\sqrt{x}\tan x}{(e^x - 1)^{3/2}}$

7+7

**4.** (a) Test the convergent of the series

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} \ldots \ldots \ldots . \infty$$

(b) Test the convergent of the series

$$x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \frac{5^5x^5}{5!} + \ldots\ldots\infty$$

7+7

**5.** (a) Obtain the Fourier series for f(x) = $e^{-x}$ in the interval $0<x<2\pi$.

(b) Express f(x) = x as a half range cosine series in $0<x<2$.

7+7

**6.** (a) Find the maximum and minimum of the function

$$f(x) = x^5 - 3x^4 + 5$$

(b) Discuss the continuity of the function

$$f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & \text{when } x \neq 0,\ y \neq 0 \\ 0 & \text{when } x = 0,\ y = 0 \end{cases}$$

7+7

**(Turn over)**

**7.** (a) Find the centre of curvature of the parabola $x=at^2$, $y=2at$ at the point ‘t’ and hence find its evolute.

(b) Expand $\tan^{-1}x$ in power of (x-1).

7+7

**8.** (a) Obtain the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

and verify that the eigen vectors are orthogonal.

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**9.** (a) Verify cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Also express $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ as a quadratic polynomial in A.

(b) If T is a liner transformation from $\mathrm{R}^3$ to $\mathrm{R}^2$ defined as

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y + z \\ y - z \end{bmatrix}$$

Determine the matrix of the liner transformation T with respect to the ordered basis.

7+7

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