$$x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \frac{5^5x^5}{5!} + \dots\infty$$

Soln- $u_n = \frac{n^n x^n}{n!}$ $\quad u_{n+1} = \frac{(n+1)^{n+1}x^{n+1}}{(n+1)!}$

$$\frac{u_n}{u_{n+1}} = \frac{n^n x^n}{n!} \times \frac{(n+1)!}{(n+1)^{n+1}x^{n+1}}$$

$$= \frac{1}{x}\left(\frac{n}{n+1}\right)^n$$

Using De-Alembart's ratio test-

$$\lim_{n\to\infty}\frac{u_n}{u_{n+1}} = \lim_{n\to\infty}\frac{1}{x}\frac{1}{(1+1/n)^n} \qquad \left[\because \lim_{n\to\infty}\frac{1}{(1+x/n)^n} = e^x\right.$$

$$= \lim_{n\to\infty}\frac{1}{xe}$$

$x < 1/e$ convergent

$x > 1/e$ divergent

$x = 1/e$ test fails

Putting $x = 1/e$ and applying logarithmic test.

$$\log\left(\frac{u_n}{u_{n+1}}\right) = \log e + n\log\left(\frac{1}{1+1/n}\right)$$

$$= 1 + n\left\{\log 1 - \log\left(1+\frac{1}{n}\right)\right\}$$

$$= 1 + n\left\{-\frac{1}{n} + \frac{1}{2n^2} - \frac{1}{3n^3} + \frac{1}{4n^4} + \dots\right\}$$

$$= \cancel{1} - \cancel{1} + \frac{1}{2n} - \frac{1}{3n^2} + \frac{1}{4n^3} + \dots$$

$$n\log\left(\frac{u_n}{u_{n+1}}\right) = \frac{1}{2} - \frac{1}{3n} + \frac{1}{4n^2} + \dots$$

$$\lim_{n\to\infty} n\log\left(\frac{u_n}{u_{n+1}}\right) = \lim_{n\to\infty}\frac{1}{2} - \frac{1}{3n} + \frac{1}{4n^2} + \dots$$

$$= \frac{1}{2} < 1$$

Divergent