To prove that the area enclosed between two Parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^3}{3}$

$x^2 = 4ay$ is $\frac{16a^3}{3}$

Given curves are

$y^2 = 4ax$ and $y^2 = 4ay$

First we have to find the area of intersection of the two curves point of intersection of the two curves are

$$\left(\frac{x^2}{4a}\right)^2 = 4ax$$

$$\left(\frac{x^2}{16a^2}\right) = 4ax \Rightarrow x^4 = 64a^3x = 0$$

$$x^4 - 64a^3x = 0$$

$$x = 0, \quad x = 4a$$

Also $y = 0, \quad y = 4a$

The Point of intersection of these 2 curves are $(0,0)$ and $(4a, 4a)$

The area of two region between the two curves

Area of the shaded region

$$\int_0^{4a} [y_2 - y_1] dx$$

$$\int_0^{4a} \left[\sqrt{4ax} - \frac{x^2}{4a}\right] dx$$

on integrating this we get

$$\left[4a^{1/2}\frac{x^{3/2}}{3}-\frac{x^3}{12a}\right]_0^{4a}$$

$$=\left[\frac{32a^2}{3}-\frac{16a^2}{3}\right]=\frac{16a^2}{3}$$

Hence Area $=\dfrac{16a^2}{3}$