

2. a) Evaluate  $\int_0^{\infty} e^{-ax} x^{m-1} \sin bx \, dx$  in terms of gamma function.

Soln.  $\int_0^{\infty} e^{-x(a-ib)} x^{m-1} \, dx$

Put  $x(a-ib) = u$

$x = \frac{u}{a-ib} \quad dx = \frac{du}{a-ib}$

$\int_0^{\infty} \frac{e^{-u} u^{m-1}}{(a-ib)^{m-1}} \frac{du}{(a-ib)} = \frac{\Gamma m}{(a-ib)^m}$

$a = r \cos \theta \quad b = r \sin \theta \quad r^2 = a^2 + b^2$

$\left(\frac{1}{a-ib}\right) \left(\frac{a+ib}{a+ib}\right) = \frac{a+ib}{a^2+b^2} = \frac{\cos \theta + i \sin \theta}{r}$

$\frac{1}{(a-ib)^m} = \frac{(\cos \theta + i \sin \theta)^m}{r^m}$

$\frac{\Gamma m}{(a-ib)^m} = \frac{\Gamma m}{r^m} (\cos m\theta + i \sin m\theta)$

$\int_0^{\infty} e^{-ax} x^{m-1} \sin bx \, dx = \frac{\Gamma m}{r^m} \sin m\theta$

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When  $a=0$ ,  $r=b$  and  $\theta = \frac{\pi}{2}$

$\int_0^{\infty} x^{m-1} \sin bx \, dx = \frac{\Gamma m}{b^m} \sin \frac{m\pi}{2}$