2.a) Evaluate $\int_0^\infty e^{-ax} x^{m-1} \sin bx \, dx$ in terms of gamma function.

Soln $\int_0^\infty e^{-x(a-ib)} x^{m-1} dx$

Put $x(a-ib) = u$

$x = \frac{u}{a-ib} \qquad dx = \frac{du}{a-ib}$

$$\int_0^\infty e^{-u} \frac{u^{m-1}}{(a-ib)^{m-1}} \frac{du}{(a-ib)} = \frac{\Gamma m}{(a-ib)^m}$$

$a = r\cos\theta \qquad b = r\sin\theta \qquad r^2 = a^2+b^2$

$$\left(\frac{1}{a-ib}\right)\left(\frac{a+ib}{a+ib}\right) = \frac{a+ib}{a^2+b^2} = \frac{\cos\theta + i\sin\theta}{r}$$

$$\frac{1}{(a-ib)^m} = \frac{(\cos\theta + i\sin\theta)^m}{r^m}$$

$$\frac{\Gamma m}{(a-ib)^m} = \frac{\Gamma m}{r^m}(\cos m\theta + i\sin m\theta)$$

$$\int_0^\infty e^{-ax} x^{m-1} \sin bx = \frac{\Gamma m}{r^m} \sin m\theta$$

When $a=0$, $r=b$ and $\theta = \frac{\pi}{2}$

$$\int_0^\infty x^{m-1} \sin bx \, dx = \frac{\Gamma m}{b^m} \sin \frac{m\pi}{2}$$